MATHEMATICS

CH-6. Application of Derivatives

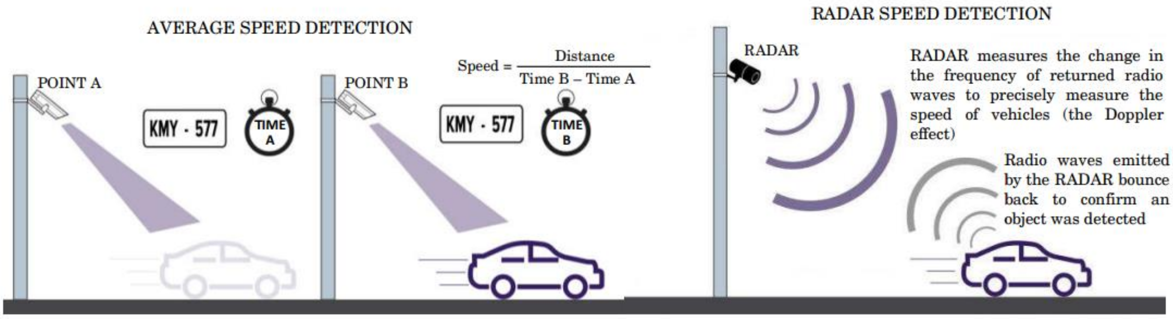
Name__________________ Date:09-08-24 Class: XII Sec: A

1.	Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.
2.	**Case Study – 1** **36.** The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight. Based on the above, answer the following questions : (i) Find the rate of growth of the plant with respect to sunlight. *1* (ii) What is the number of days it will take for the plant to grow to the maximum height ? *2* (iii) What is the maximum height of the plant ? *1*
3.	Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if (A) $f'(x) < 0, \forall x \in (a, b)$ (B) $f'(x) > 0, \forall x \in (a, b)$ (C) $f'(x) = 0, \forall x \in (a, b)$
4.	Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

5.

Case Study – 1

36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. *1*

(ii) Find $\frac{d\theta}{dx}$. *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. *2*

OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. *2*

6. Find the absolute maximum and minimum values of the function $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [0, 1]$.

7. Find local maximum value and local minimum value (whichever exists) for the function $f(x) = 4x^2 + \frac{1}{x}$ $(x \neq 0)$.

8. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

9. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

10. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

11. Show that :

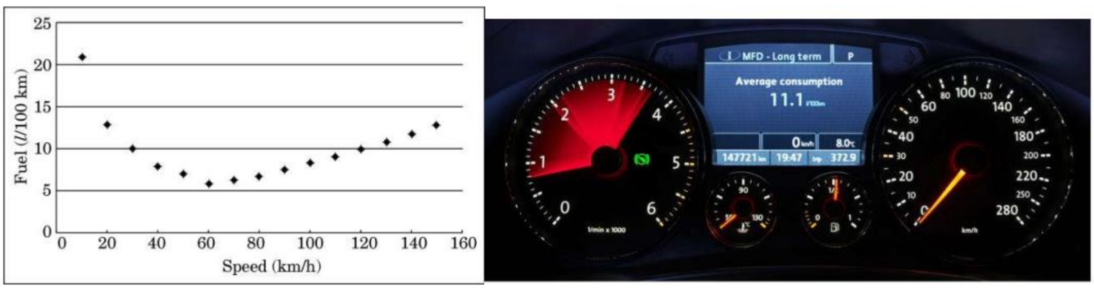
$$\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$$

, Also find the maximimum value and minimum value of the function.

12.

Case Study – 1

Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\frac{dF}{dV}$. *1*

	(iii) (a) Find the speed V for which fuel consumption F is minimum. **OR** (iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$.
13.	Determine whether the function $f(x) = x^2 - 6x + 3$ is increasing or decreasing in [4, 6].
14.	If the sides of a square are decreasing at the rate of 1·5 cm/s, the rate of decrease of its perimeter is : (A) 1·5 cm/s (B) 6 cm/s (C) 3 cm/s (D) 2·25 cm/s
15.	Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.
16.	Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval [1, 2].
17.	It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.
18.	The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.
19.	The point of inflexion of a function f(x) is the point where (A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point. (B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point. (C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point. (D) $f'(x) \neq 0$.
20.	The function $f(x) = kx - \sin x$ is strictly increasing for (A) $k > 1$ (B) $k < 1$ (C) $k > -1$ (D) $k < -1$